

## 2.10 Setting out angles—use of set squares

The setting out of angles using a protractor was introduced in Fig. 2.37 of *Basic Engineering*. However, angles that are multiples of  $15^\circ$  can be set out quickly and accurately using a  $45^\circ$  set square and a  $60^\circ$  set square. Figure 2.26 shows how these set squares can be combined together to produce a range of angles.

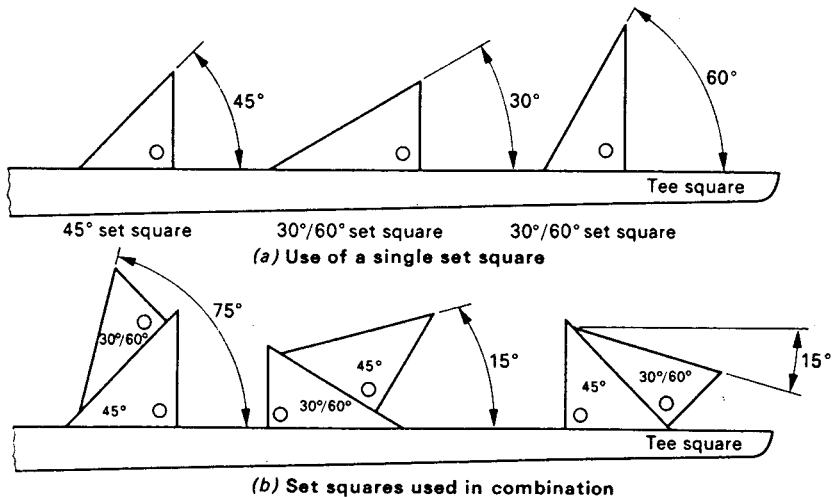
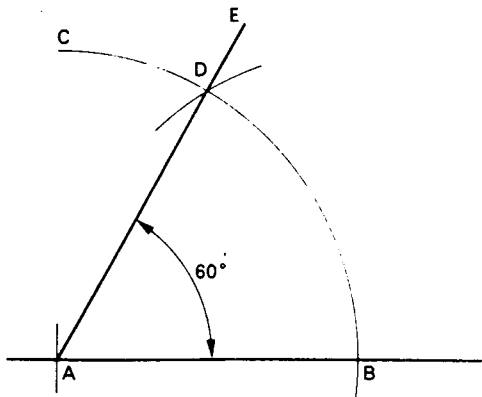


Fig.2.26 Construction of angles using set squares

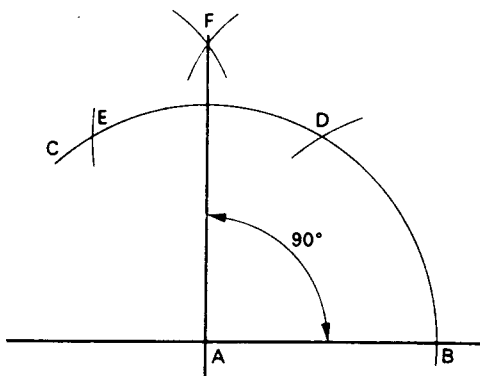
## 2.11 Setting out angles—use of compasses

Angles which are multiples of  $7\frac{1}{2}^\circ$  can be set out very accurately using compasses alone. The basic angle from which all the others can be derived is the  $60^\circ$  angle shown in Fig. 2.27(a). The constructions for  $90^\circ$  and  $45^\circ$  angles are shown at (b) and (c), respectively, in Fig. 2.27.



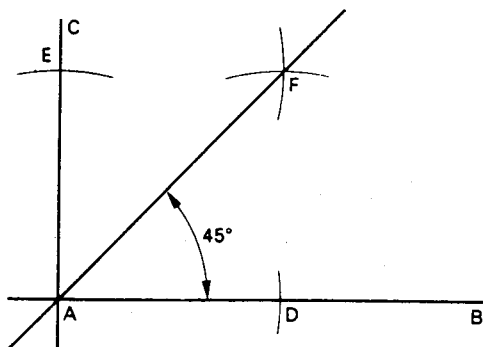
1. Let A be the apex of the angle
2. With centre A draw an arc BC of large radius
3. Step off BD equal in radius to AB
4. Draw a line AE through D
5. The angle EAB is  $60^\circ$

(a) Construction of a  $60^\circ$  angle



1. Let A be the apex of the angle
2. With centre A draw an arc BC of large radius
3. Step off BD and DE equal in radius to AB
4. With centre D draw any arc F
5. With centre E draw an arc equal in radius to DF
6. Join AF with a straight line
7. Angle BAF is  $90^\circ$

(b) Construction of a  $90^\circ$  angle

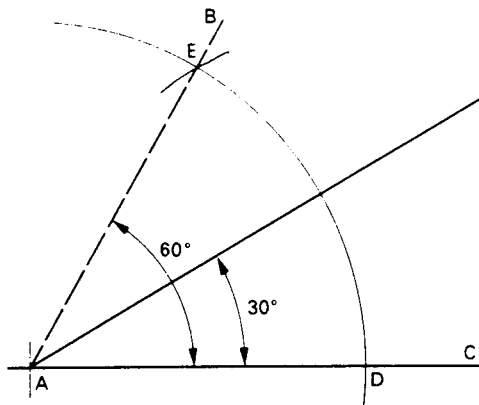


1. Draw AB and AC at right angles ( $90^\circ$ ) to each other as described in Fig. 2.29 (b)
2. With centre A, and with large radius, draw an arc to cut AB at D and AC at E
3. With centres E and D draw arcs of equal radius to intersect at F
4. Draw a straight line from A through F
5. Angle BAF is  $45^\circ$   
(AF bisects – halves  $\angle BAC$ )

(c) Construction of a  $45^\circ$  angle

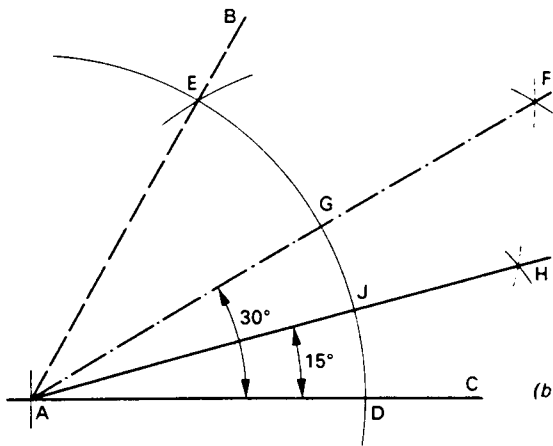
Fig.2.27 Construction of angles using compasses

The bisection of the  $60^\circ$  angle to produce  $30^\circ$  and the successive bisection of this angle to produce  $15^\circ$  which is bisected to produce  $7\frac{1}{2}^\circ$  is shown in Fig. 2.28.



1. Draw AB and AC at  $60^\circ$  to each other as described in Fig 2.29 (a)
2. With centre A, and a large radius, draw an arc to cut AB at E and AC at D
3. With centres E and D draw arcs of equal radius to intersect at F
4. Draw a line from A through F
5. Angle CAF is  $30^\circ$  (half  $60^\circ$ )

(a) Construction of a  $30^\circ$  angle



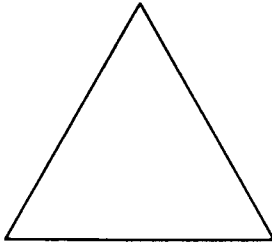
1. Draw AC and AF at  $30^\circ$  to each other as described in Fig. 2.30 (a)
2. With centres G and D draw arcs of equal radius to intersect at H
3. Draw a line from A through H
4. Angle CAH is  $15^\circ$  (half  $30^\circ$ )
5. With centres J and D a further bisection can be made as described in 2, 3 and 4 above. This would give angle of  $7\frac{1}{2}^\circ$

(b) Construction of angles of  $15^\circ$  and  $7\frac{1}{2}^\circ$

Fig.2.28 Bisection of angles

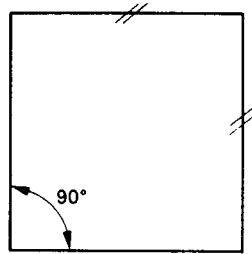
## 2.12 Construction of plane figures

All the regular plane figures met with in fabrication engineering can be set out by geometrical constructions, using compasses and set squares. Some of these figures, which are to be constructed in this section of the chapter, are described in Fig. 2.29.



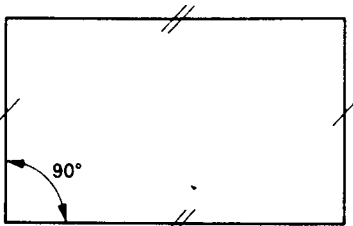
**Triangle**

*A plane figure with 3 sides*



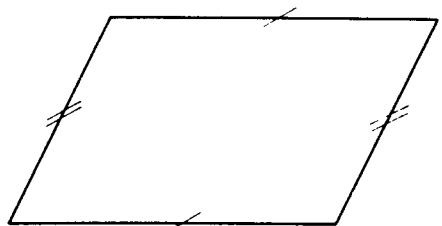
**Square**

*A plane figure with 4 sides of equal length and at right angles ( $90^\circ$ ) to each other*



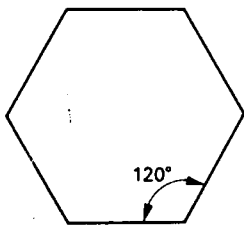
**Rectangle**

*A plane figure with 4 sides at  $90^\circ$  to each other. Only opposite sides are of equal length*



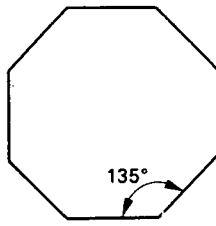
**Parallelogram**

*A plane figure with 4 sides. Opposite sides are parallel to each other and the same length. Adjacent sides are not at right angles*



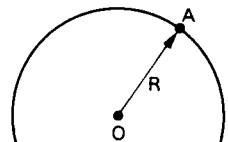
**Hexagon**

*A plane figure with 6 sides of equal length. Adjacent sides are at  $120^\circ$  to each other*



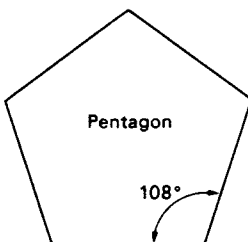
**Octagon**

*A plane figure with 8 sides of equal length. Adjacent sides are at  $135^\circ$  to each other*



**Circle**

*The plane figure produced when a point 'A' moves at a constant distance 'R' about a point 'O'*



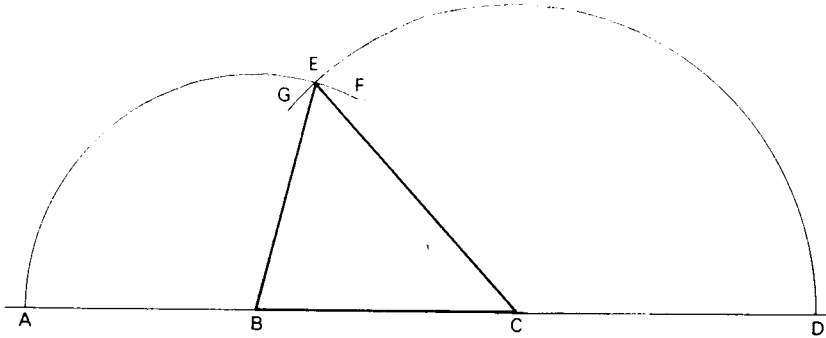
**Pentagon**

*A regular polygon with five sides of equal length. Adjacent sides are  $108^\circ$  to each other*

**Fig. 2.29 Some regular plane figures**

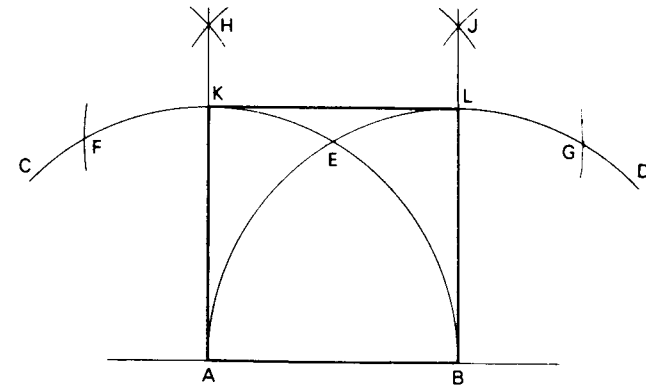
The basic constructions associated with these figures are given in the following illustrations:

- |                  |   |            |
|------------------|---|------------|
| 1. Triangle      | — | Fig. 2.30. |
| 2. Square        | — | Fig. 2.31. |
| 3. Rectangle     | — | Fig. 2.32. |
| 4. Parallelogram | — | Fig. 2.33. |
| 5. Pentagon      | — | Fig. 2.34. |
| 6. Hexagon       | — | Fig. 2.35. |
| 7. Octagon       | — | Fig. 2.36. |
| 8. Circle        | — | Fig. 2.37. |



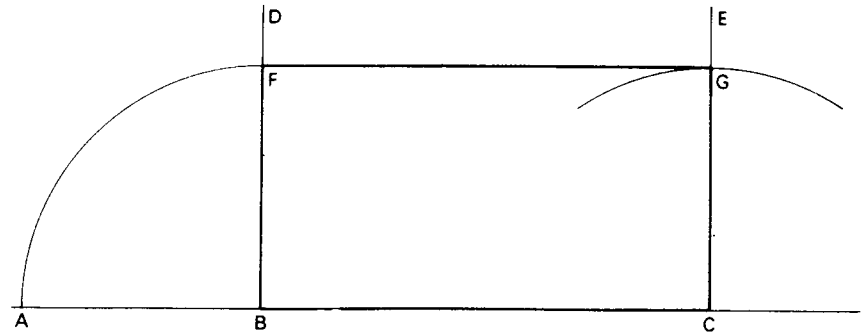
1. Draw AB, BC and CD equal in length to the sides of the required triangle
2. With centre B and radius AB draw the arc AF
3. With centre C and radius CD draw the arc DG
4. Where the arcs intersect at E is an apex of the triangle
5. Join BE and CE with straight lines to form the triangle BCE

**Fig.2.30 To construct a triangle**

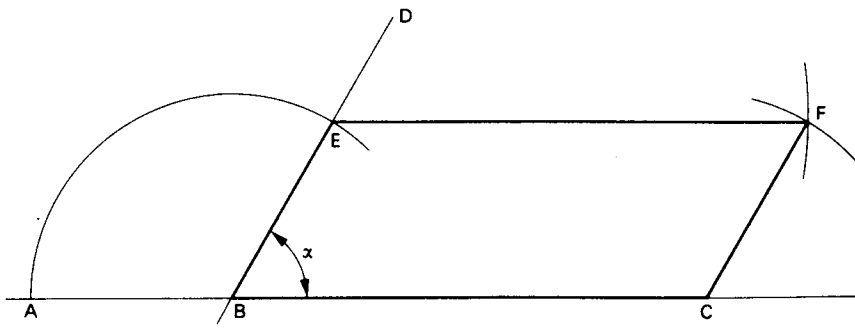


1. Mark off one side of the square AB on the base line
2. With centre 'A' and radius AB draw the arc BC
3. With centre 'B' and radius AB draw the arc AD
4. With centre 'E' and radius AB step off 'F' and 'G' on arcs BC and AD respectively
5. With centres 'E' and 'F' draw arcs of equal radius to intersect at H
6. With centres 'E' and 'G' draw arcs of equal radius to intersect at J
7. Erect perpendiculars AH and BJ
8. The arcs BC and AD cut the perpendiculars AH and BJ at 'K' and 'L' respectively
9. To complete the square join 'K' and 'L'

**Fig.2.31 To construct a square**

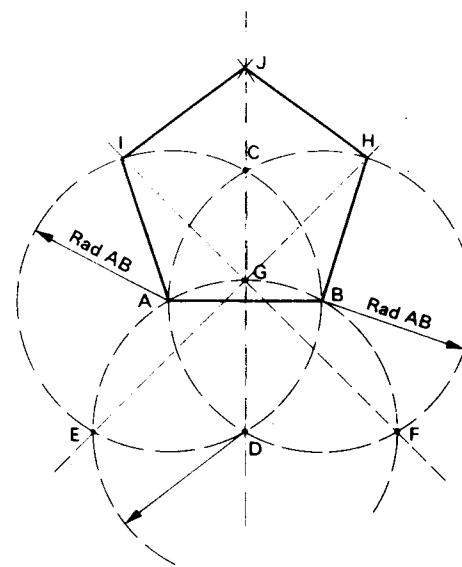


1. Mark off the lengths of a short side AB and a long side BC on the base line
2. Using the constructions demonstrated in the previous examples, or using a set square for simplicity, erect perpendiculars BD and CE at 'B' and 'C'
3. With centre 'B' and radius AB draw an arc to cut BD at 'F'
4. With centre 'C' and radius AB draw an arc to cut CE at 'G'
5. To complete the rectangle join F and G with a straight line



1. Mark off one short side AB and one long side BC on the base line
2. Draw BD at the required angle  $\alpha$  to the base line using an adjustable set square or protractor
3. With centre 'B' and radius AB draw an arc to cut BD at 'E'
4. With centre 'C' and radius AB draw an arc
5. With centre 'E' and radius BC draw an arc to cut the arc drawn in (4) at F
6. To complete the parallelogram join EF and CF with straight lines

Fig.2.33 To construct a parallelogram

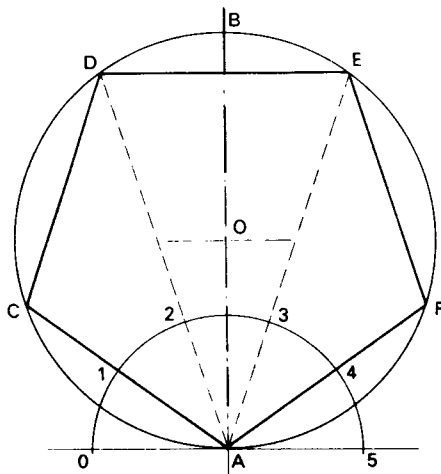


(b) To construct a pentagon given the length of one side

Fig 2.34 Two methods of constructing a pentagon

Method of construction

1. Draw the given side AB
2. With centres A and B and compasses set to radius AB, draw two circles to intersect at C and D
3. Join CD
4. With centre D and radius AB, draw an arc to cut the two previously drawn circles at E and F, and CD at G
5. From E and F draw lines through point G to cut the two circles at H and I
6. With radius AB and centres H and I draw arcs to intersect at J on DC produced
7. Join AIJHB to complete the pentagon

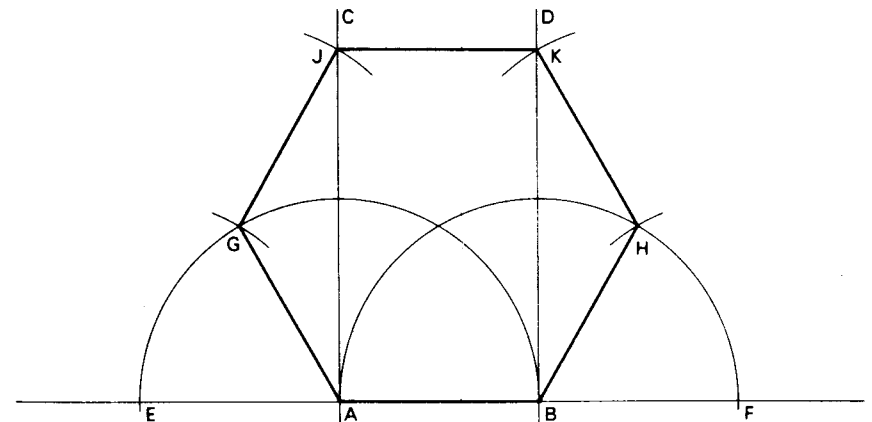


Method of construction

1. With centre O and compasses set to required radius draw the given circle
2. Draw a vertical diameter AB and a tangent at A.
3. With centre A and any suitable radius draw a semi-circle
4. With the compass divide the semi-circle into five equal parts (ie. same number as sides in the pentagon) and number 0-5
5. From tangent point A draw lines through the points 1, 2, 3 and 4 on the semi-circle to cut the given circle, at C, D, E and F
6. B Complete the pentagon by joining CDEF

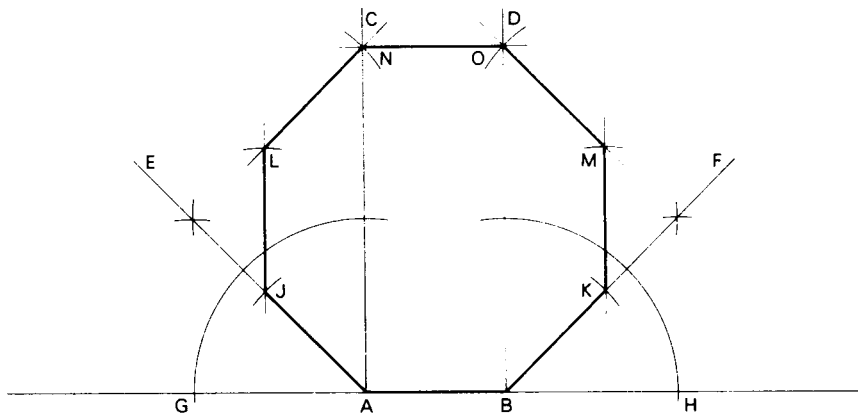
**Note:** This method of construction may be used to construct any regular within a circle. The semi-circle is divided into the same number of equal parts as there are sides in the polygon

(a) To construct a pentagon in a given circle



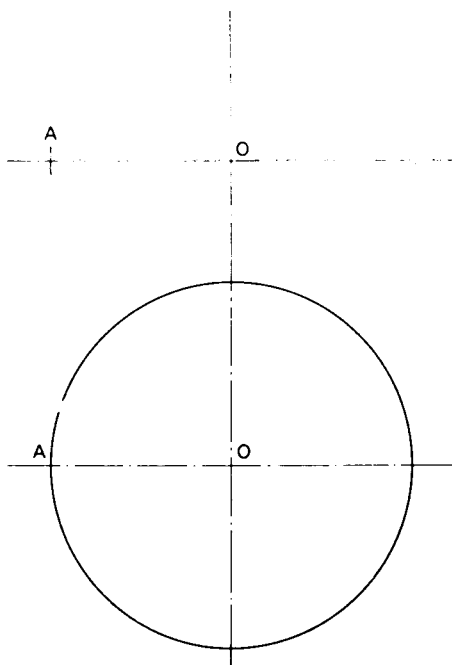
1. Mark off one side AB on the base line
2. Erect perpendiculars AC and BD using one of the previously demonstrated constructions or set square
3. With centre 'A' and radius AB draw an arc to cut the base line at 'E'
4. With centre 'E' and radius AB draw an arc to cut the previous arc (3) at 'G'
5. With centre 'B' and radius AB draw an arc to cut the base line at 'F'
6. With centre 'F' and radius AB draw an arc to cut the previous arc (5) at 'H'
7. With centre 'G' and radius AB draw an arc to cut the perpendicular AC at 'J'
8. With centre 'H' and radius AB draw an arc to cut the perpendicular BD at 'K'
9. To complete the hexagon join AG, GJ, JK, KH and HB with straight lines

Fig.2.35 To construct a hexagon



1. Mark off the length of one side AB on the base line
2. Erect perpendiculars AC and BD from 'A' and 'B' respectively
3. Bisect the angle CAG so that EA lies at  $45^\circ$  to GA
4. Similarly, bisect the angle DBH so that BF lies at  $45^\circ$  to BH
5. With centre 'A' and radius AB draw an arc to cut AE at J
6. With centre 'B' and radius AB draw an arc to cut BF at K
7. Erect perpendiculars at 'J' and 'K'
8. With centres 'J' and 'K' and radius AB draw arcs to cut the perpendiculars at L and M respectively
9. With centres 'L' and 'M' and radius AB draw arcs to cut AC and BD at 'N' and 'O' respectively
10. To complete the octagon join AJ, JL, LN, NO, OM, MK and KB with straight lines

**Fig. 2.36 To construct an octagon**



1. Locate the centre of the hole by the intersection 'O' of two lines as near to  $90^\circ$  as possible
2. Mark the radius of the circle OA on one of the centre lines
3. With centre 'O' and radius OA draw the circle

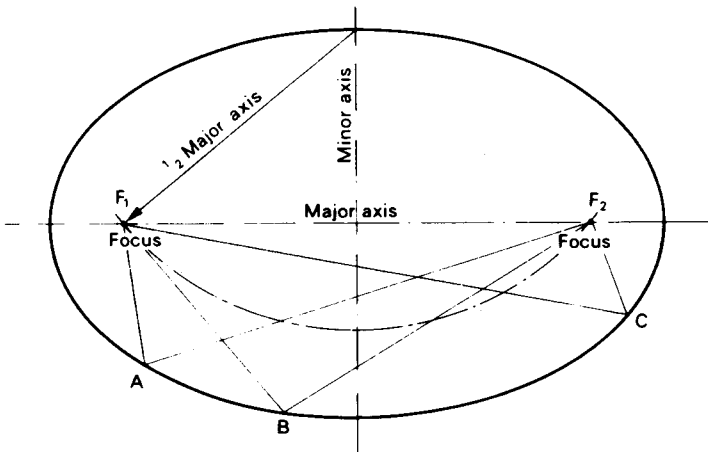
**Fig. 2.37 To construct a circle**

## 2.13 The construction of ovals and ellipses

Ovals and ellipses are common shapes encountered in sheet metal work. The essential differences between these two figures will now be explained.

The oval differs from the true ellipse in the fact that *the oval is an approximate ellipse drawn with compasses*, and is therefore a construction made up of arcs of circles. *Because no part of a true ellipse is part of a circle it cannot be drawn with compasses.*

The shape of an ellipse can be traced out by a point which moves so that the sum of its distances from two fixed points on the major axis is always constant. These fixed points are called the 'foci' or 'focal points' of the ellipse. This fundamental principle is illustrated in Fig. 2.38.



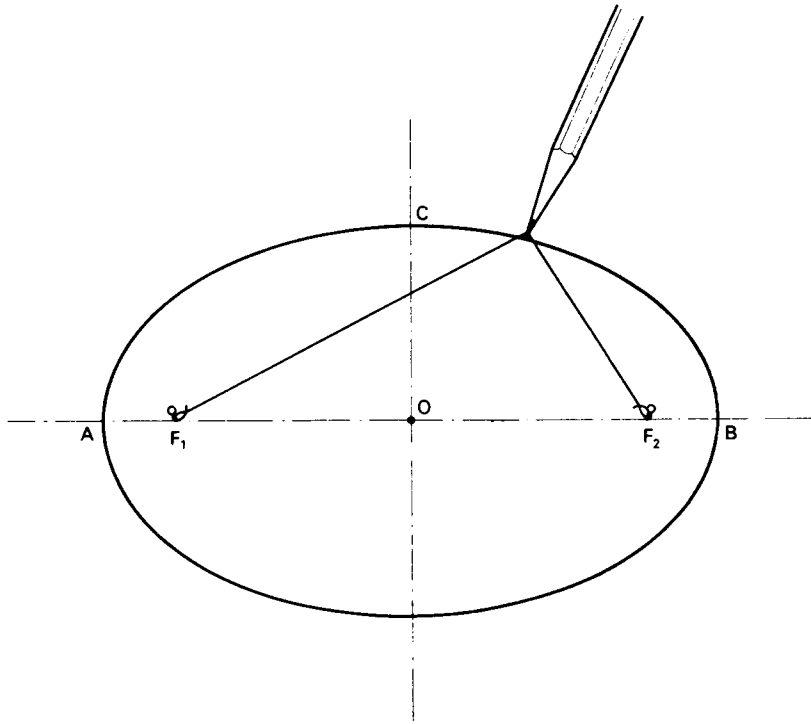
**Note:** Lines  $F_1A F_2$ ,  $F_1B F_2$  and  $F_1C F_2$  are all the same length

**Fig 2.38 Principle of the ellipse**

The ELLIPSE is symmetrical about two AXES which bisect each other at  $90^\circ$ . The longer 'axis' is termed the MAJOR AXIS (or major diameter), and the shorter 'axis' is termed the MINOR AXIS (or minor diameter).

The path of an ellipse may be traced out by a point which moves from two fixed points on the major axis (termed FOCI) in such a way that the sum of its distances from the foci is always constant.

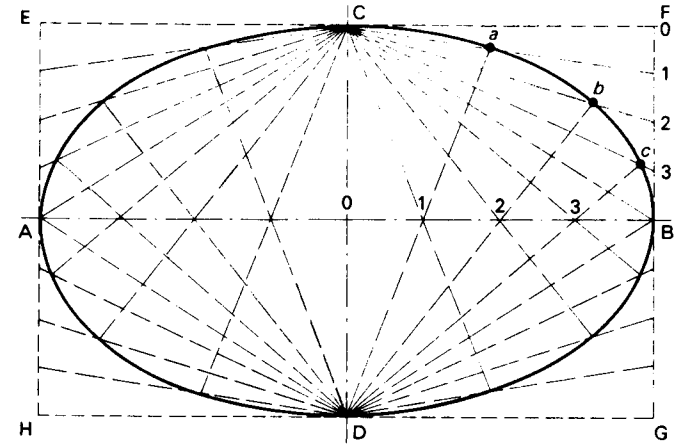




**Fig 2.39 Practical method of drawing an ellipse**

*Method of construction*

1. Draw the major and minor axes.
2. With centre C and radius equal to A O ( $\frac{1}{2}$  major axis) draw arcs to locate the focal points  $F_1$  and  $F_2$  on the major axis A B.
3. Fix pins in the focus points. Make two loops in a piece of cotton such that its length (including loops) is exactly equal to the major axis. Slip the loops over the locating pins.
4. Place the pencil point so that the cotton is stretched taut, as shown in the diagram. The path of the ellipse may now be traced by moving the pencil point whilst keeping the cotton taut.



**Fig 2.40 Construction of an ellipse - rectangle method**

*Method of construction*

1. Draw the major and minor axis A B and C D.
2. Draw the rectangle E F G H whose sides are equal to the length of the axis.
3. Divide the major axis A B into any equal number of parts, and divide the ends of the rectangle (E H and F G) into the same number of equal parts.

**Note:** For clarity only one quarter of the rectangle has been numbered.

4. From each end of the minor axis C D draw radial lines (as shown opposite), into points 1, 2 and 3.
5. Draw radial lines from C and D through the points 1, 2 and 3 on the major axis A B these will intersect the previous radial lines at points a, b and c.

- The outline of the ellipse may now be drawn by joining the location points between A C, C B, B D and D A.

#### ELLIPSE CONSTRUCTION – CONCENTRIC CIRCLE METHOD

Figure 2.41 shows how an ellipse may be constructed by the concentric circle method.

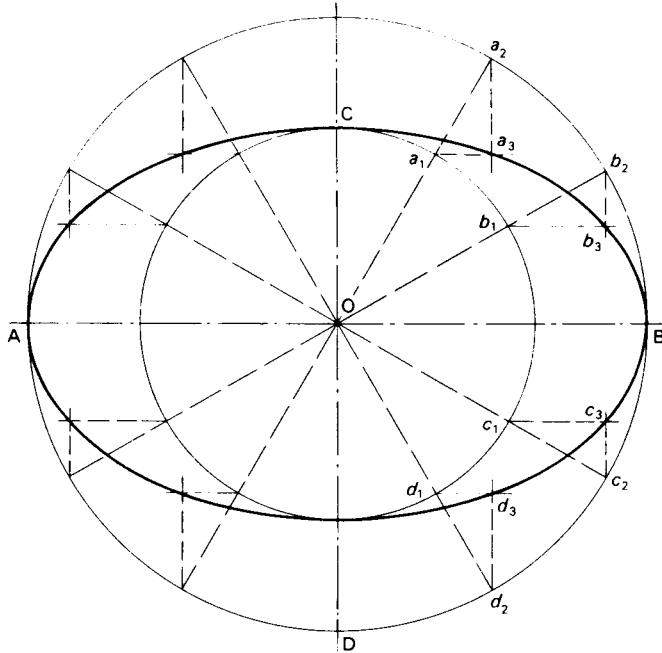


Fig 2.41 Construction of an ellipses - concentric circle method

#### Method of construction

- Draw the major and minor axis, A B and C D. These intersect at O.
- Using each axis as a diameter, with centre O, draw two CONCENTRIC CIRCLES as shown.
- Using a 60°/30° set square in conjunction with a Tee square draw four diameters as indicated by a, b, c and d, on the minor circle and a<sub>2</sub>, b<sub>2</sub>, c<sub>2</sub> and d<sub>2</sub>, on the major circle. From a<sub>2</sub>, b<sub>2</sub>, c<sub>2</sub> and d<sub>2</sub>, draw vertical lines towards the major axis A B.
- From a, b, c and d, draw horizontal lines towards the minor axis C D.
- These lines will intersect and locate points on the ellipse at a<sub>3</sub>, b<sub>3</sub>, c<sub>3</sub> and d<sub>3</sub>.

- The outline of the ellipse may now be drawn by joining the location points between A C, C B, B D and D A.

#### PLOTTING AN ELLIPSE USING TRAMMELS

Figure 2.42 shows how an ellipse may be plotted using trammels.

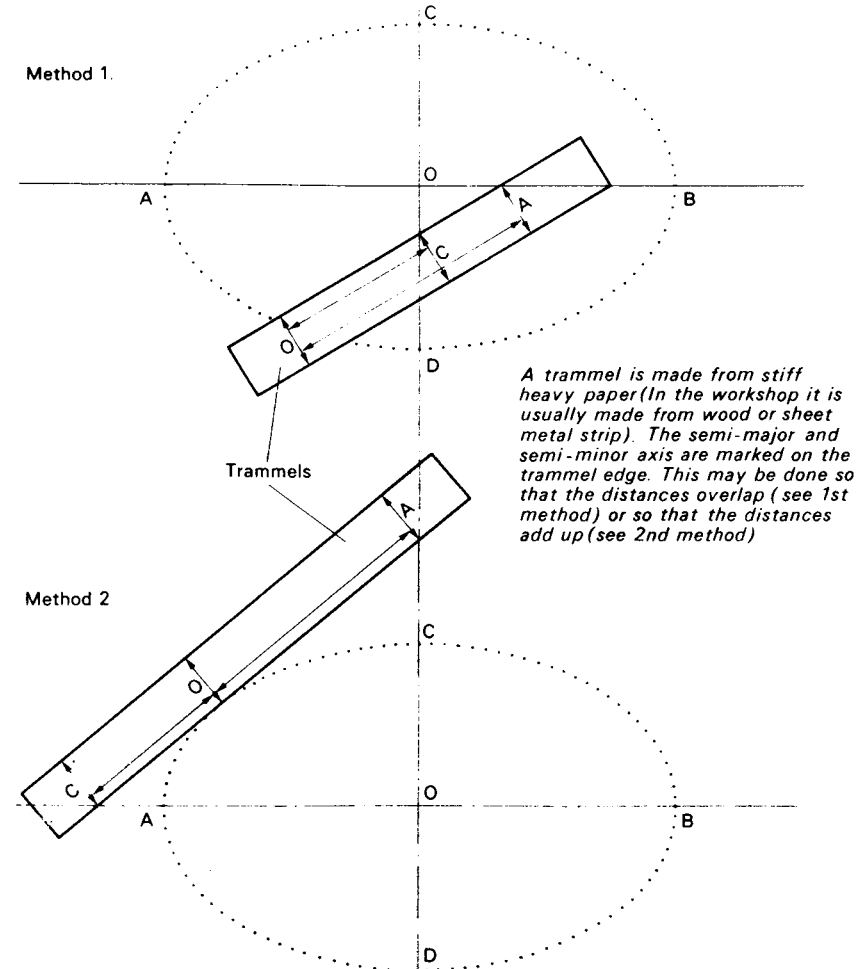


Fig.2.42 The plotting of an ellipse using trammels

#### 1st method

Mark the paper trammel such that the distance A O = ½ major axis A B and the distance C O = ½ minor axis C D, these distances overlapping having a common starting point O.



### Method of construction

1. Draw the major and minor axis A B and C D. These intersect at O.
2. Using a 60°/30° set square in conjunction with a Tee square draw an EQUILATERAL TRIANGLE with sides equal to  $\frac{1}{2}$  the major axis, as shown at A E O.
3. With compasses set to  $\frac{1}{2}$  the minor axis and centre O, draw an arc from C to cut side E O of the triangle at F.
4. Draw a line from C through F to cut side E A of the triangle at G.
5. Using the 60°/30° set square draw a line from G parallel to E O to cut the major axis A B at I and the minor axis C D produced at H.
6. H and I are the centres for drawing arcs to produce the approximate ellipse (oval). For corresponding centres, mark B K = A I and O J = O H using compasses. Curve G C is drawn with H as centre. Curve G A is drawn with I as centre.

## 2.14 Further useful geometric constructions

It will be explained in Chapter 5 that geometric constructions can be used to simplify the marking out of geometric components. In addition to the plane figures just described (2.12 and 2.13) the following constructions are also of great use to the craftsman.

### 1 PARALLEL LINES

Figure 2.45 shows how parallel lines may be drawn.

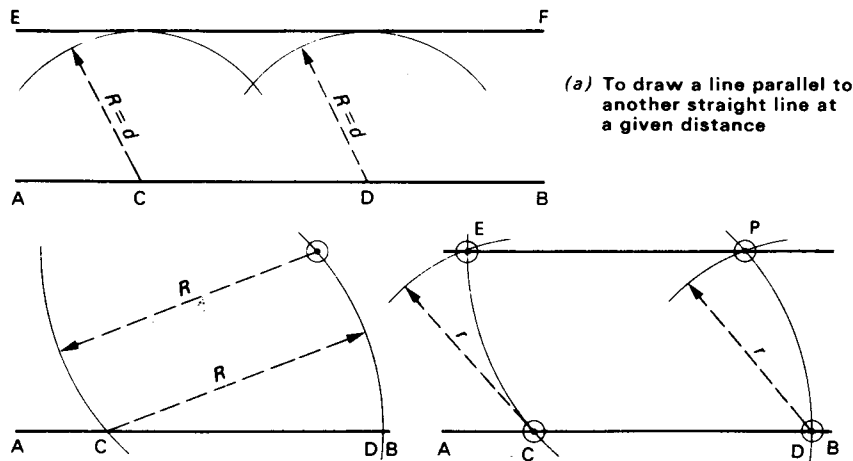


Fig 2.45 Construction of parallel lines

It will be seen in Fig. 2.45(a) that to draw a line parallel to another straight line at a given distance the following construction should be used.

With radius  $R$  (= to required distance) and any two points C and D on the given line A B, draw arcs. Draw the line E F tangential to the two arcs.

It will be seen in Fig. 2.45(b) that to draw a line parallel to a given straight line from a point outside it the following construction should be used.

With the centre at the point P, and any radius  $R$  describe an arc to locate a point C on the given line A B. With centre C and the same radius  $R$  draw an arc to locate a point D on the given line A B. With centre C and radius  $r$  (= D P) draw an arc to intersect the arc radius  $R$  at E. Draw a line through P and E.

### 2 TO LOCATE THE CENTRE OF AN ARC OR A CIRCLE

This construction is given in Fig. 2.46. Select three points C, D and E on the arc A B. Bisect the arc lengths between these points, using C, D and E as centres. The point O where the bisectors intersect will be the centre of the given arc or circle.

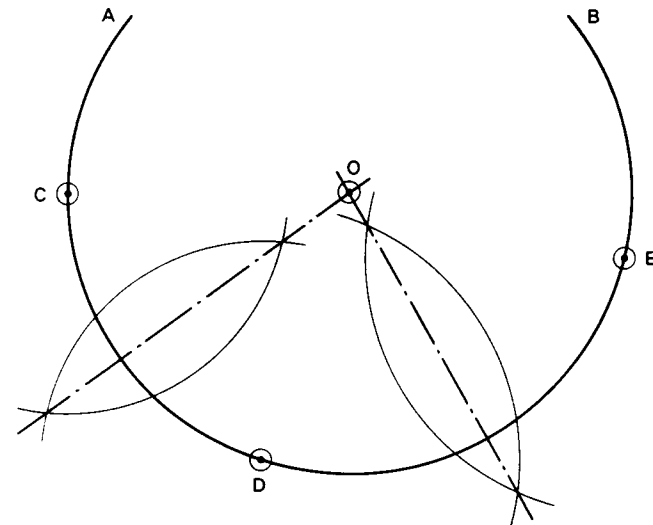


Fig 2.46 To locate the centre of a given arc or circle

### 3 DIVISION OF A STRAIGHT LINE

The construction for dividing a straight line into a number of equal or unequal parts was given in Chapter 2 of *Basic Engineering*.

#### 4 TO CONSTRUCT A CIRCLE INSIDE OR OUTSIDE A GIVEN TRIANGLE

Figure 2.47(a) shows how a circle is drawn inside (*inscribed*) a triangle. The method is to bisect any two angles (Fig. 2.28). The point O where the bisectors intersect is the centre of the circle which is tangential to all three sides of the given triangle.

Figure 2.47(b) shows how a circle is drawn outside (*circumscribed*) a triangle. The method is to bisect any two sides of the given triangle. The point O where the two bisectors intersect is the centre of the circle which may be drawn to pass the points A, B and C.

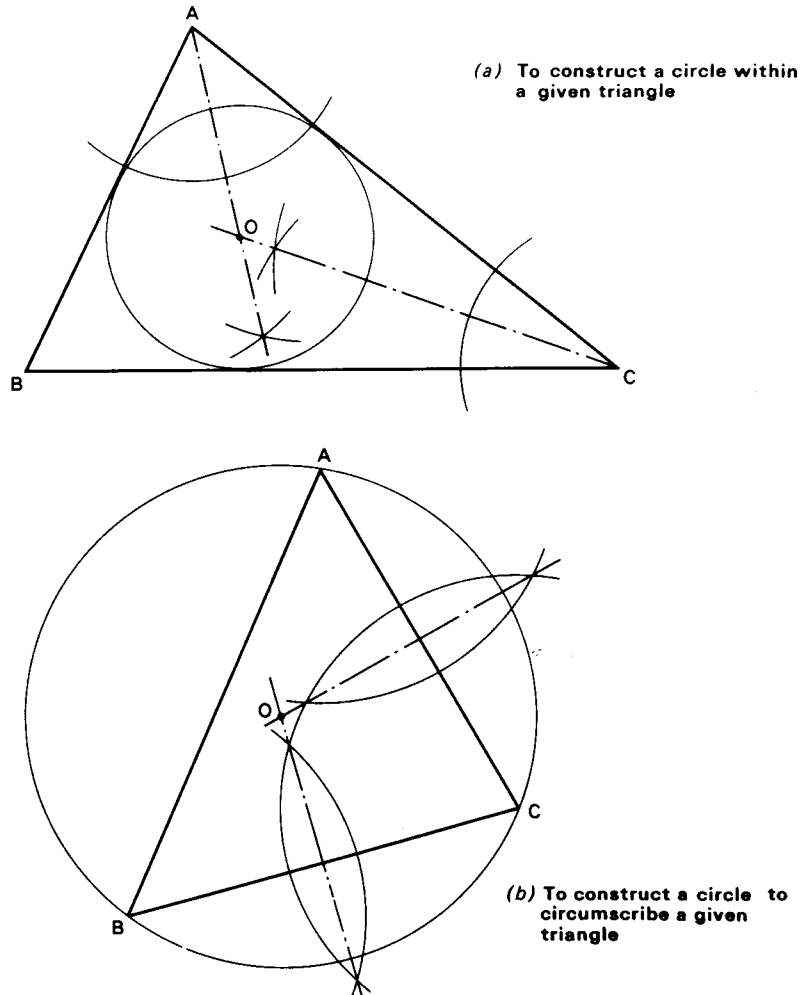


Fig. 2.47 Inscribed and circumscribed triangles

#### 5 ARCS OF CIRCLES

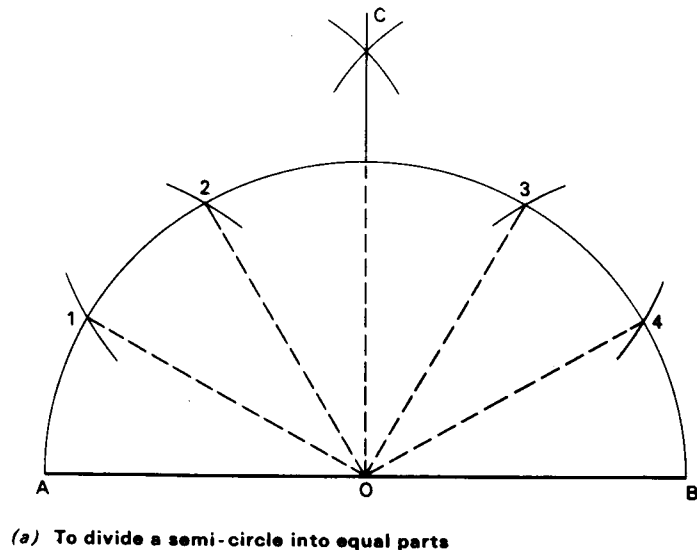
Figure 2.48 shows various constructions associated with arcs of circles. Figure 2.48(a) shows how to divide a semi-circle into a number of equal parts.

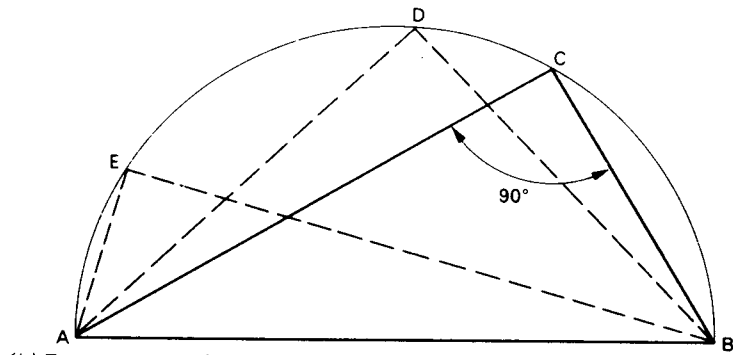
Bisect the DIAMETER A C. With a radius equal to HALF THE DIAMETER, and centres A, D and C draw arcs to cut the semi-circular circumference at points 1, 2, 3 and 4. The semi-circle is now divided into SIX EQUAL PARTS.

Note: This construction is a basic method used in pattern development for cylindrical and conical fabricated articles in sheet metal and plate work. It is also used for pyramids with HEXAGONAL bases.

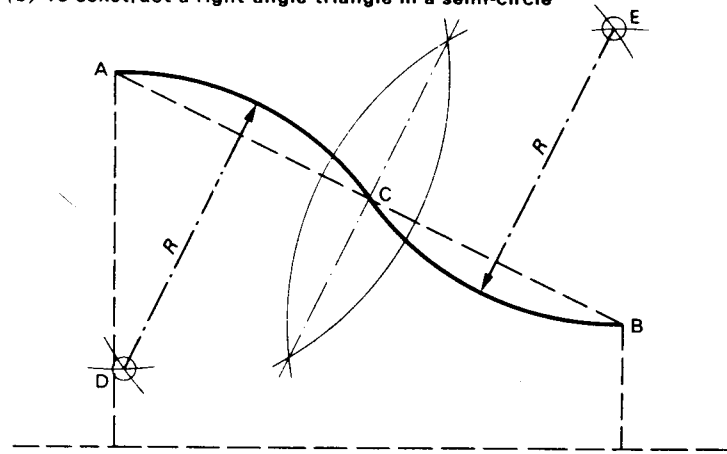
Figure 2.48(b) shows how to construct a right-angle in a semi-circle. Select any point C on the circumference of the given semi-circle and draw lines from the DIAMETER to termination points A and B to C. Then the angle ACB is  $90^\circ$ . Similarly the angles at E and D are right-angles.

Figure 2.48(c) shows how to construct an OGEE curve. The termination points of the curve A B are normally given dimensionally. Join A and B by a straight line and bisect it at C. With radius R equal to the required radius of the curve, and with points A, C and B as centres draw arcs to locate D and E. With the same radius and centres D and E complete the curve.





(b) To construct a right-angle triangle in a semi-circle



(c) To construct an ogee curve

Fig. 2.48 Constructions associated with arcs of circles

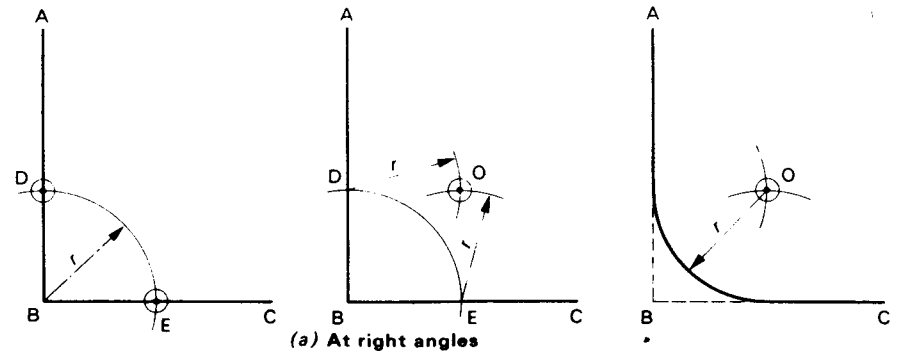
## 6 TANGENCY (STRAIGHT LINE)

Figure 2.49 shows how an arc may be drawn to touch two straight lines. Figure 2.49(a) shows the construction for drawing an arc tangential to two perpendicular lines. With the given radius  $r$  and centre B locate points E and D. With D and E as centres, and the same radius draw arcs to intersect at point O. With the same radius and centre O draw the given arc to touch the two straight lines.

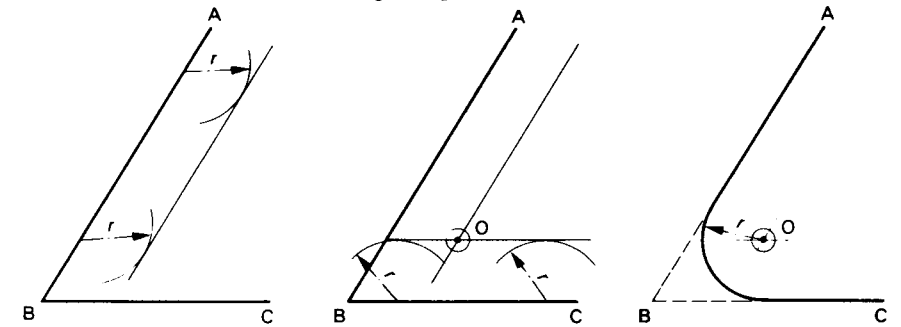
Figure 2.49(c) shows the construction for drawing an arc tangential to two lines forming an obtuse angle.

At a distance equal to the given radius  $r$ , construct parallel lines to AB and BC to intersect at a point O.

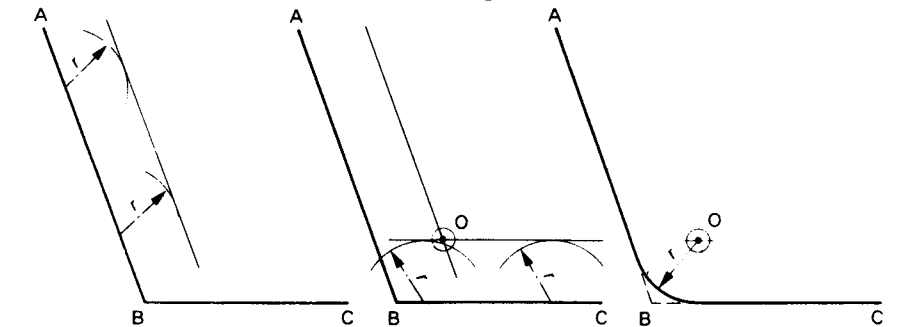
With centre O and the given radius  $r$ , draw the required arc to touch the two straight lines.



(a) At right angles



(b) At an acute angle



(c) At an obtuse angle

Fig. 2.49 To draw an arc of given radius to touch two straight lines

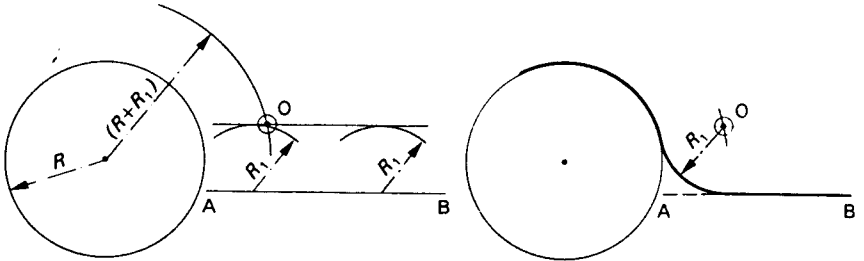
## 7 TANGENTIAL ARCS

Figure 2.50 shows the constructions associated with tangential arcs. Figure 2.50(a) shows the construction for an arc that is tangential to a circle and a straight line. Construct a parallel line, at a distance equal to the given radius  $R_1$ , to the given straight line AB. Add  $R_1$  to the radius  $R$  of the given circle and from the centre of the circle draw an arc to locate point O. With centre O and radius  $R_1$  draw the required arc to touch the straight line and the circle.

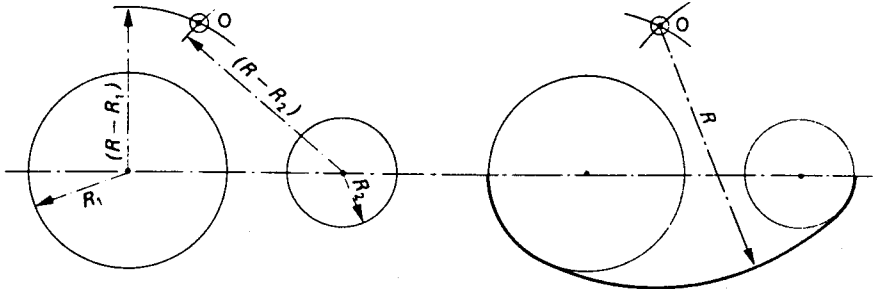
Figure 2.50(b) shows the construction of an external, tangential arc. Let  $R$  represent the radius of the given arc, and  $R_1$  and  $R_2$  the radii of the circles it is required to touch. Subtract the circle radii from the given radius of the arc, in turn, and from the respective centres of the circles draw arcs to intersect at point  $O$ . With  $O$  as centre and radius  $R$  draw the required arc to touch the circles externally.

Figure 2.50(c) shows the construction of an internal, tangential arc.

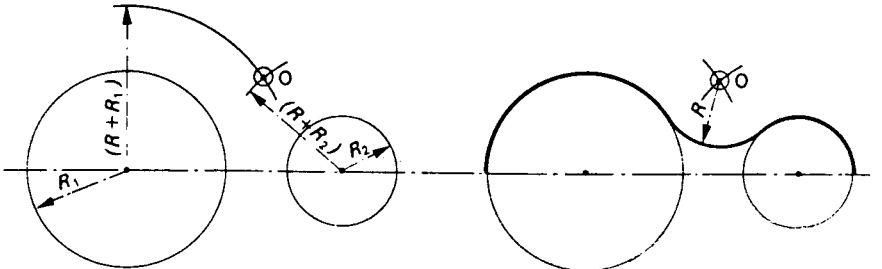
Add the circle radii to the given radius of the arc  $R$ . From the respective centres of the circles draw arcs to intersect at a point  $O$ . With  $O$  as centre and radius  $R$  draw the required arc to touch the circles internally.



(a) To draw an arc of given radius to touch a circle and a straight line



(b) To draw an arc of given radius to touch two circles externally



(c) To draw an arc of given radius to touch two circles internally

Fig. 2.50 Tangential arcs

## 2.15 Development of surfaces

The three basic methods used in pattern development are:

1. The PARALLEL LINE method.
2. The RADIAL LINE method.
3. The TRIANGULATION method.

In this book the principles of these methods will be explained, and the examples used will be limited to right prisms, right pyramids, the right cylinder, and right cone, and their frustums between parallel planes. *The development of a surface is the unrolling or unfolding of that surface so that it lies in one plane.*

The faces of *Prisms* are planes with their edges parallel. The unfolding of these faces will produce a development which takes the form of a simple rectangle.

*A cylinder* is developed by unrolling its surface, thus producing a rectangle having one side equal in length to the 'circumference' of the cylinder, the other side being equal to the length or height of the cylinder.

*A pyramid*, when its surface is unfolded, forms a development which consists basically of a number of triangles. *The base of each triangle is equal to the length of the base of the pyramid. The sides of each triangle are equal in length to the 'slant' edges of the pyramid.*

*A cone*, is developed by unrolling its surface. The circular base of the cone unrolls around a point, which is the apex of the cone, for a distance equal to its circumference. *The radius of the arc producing the base of the development is equal to the 'slant height' of the cone.*

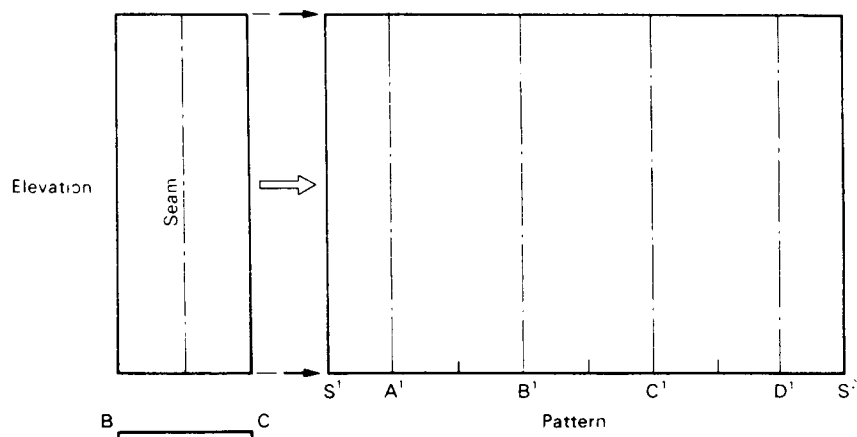
In practice a complete cone is rarely required, except perhaps 'flat cones' which are used as 'caps', for example, on stove pipes. However, in the fabrication industry conical sections are constantly required to be manufactured. These components are part cones, often referred to as TRUNCATED CONES. When the cone is cut off parallel with its base, i.e. the top portion removed, the remaining portion is called the FRUSTUM.

## 2.16 Parallel line developments

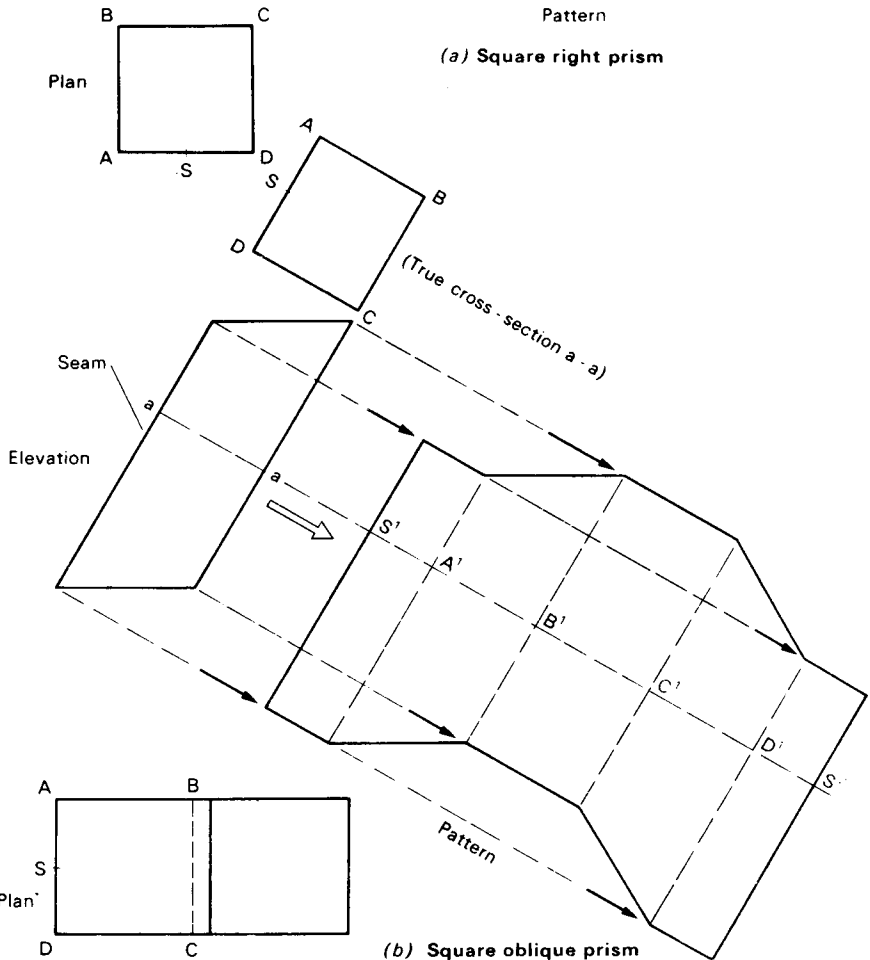
The 'parallel line method' of pattern development depends upon a principle of locating the shape of the pattern on a series of parallel lines. All articles or components which belong to the class of PRISMS, *which have a constant 'cross-section' throughout their length, may be developed by the parallel line method.*

Elementary examples of parallel line development are illustrated in Fig. 2.51 to 2.53 inclusive.



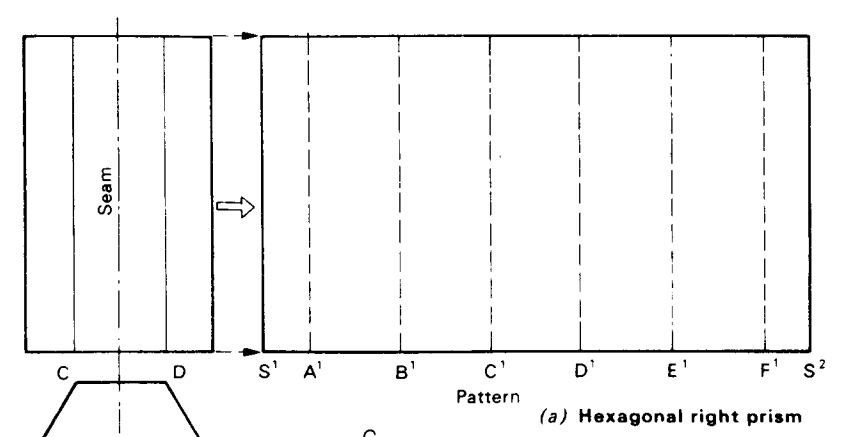


(a) Square right prism

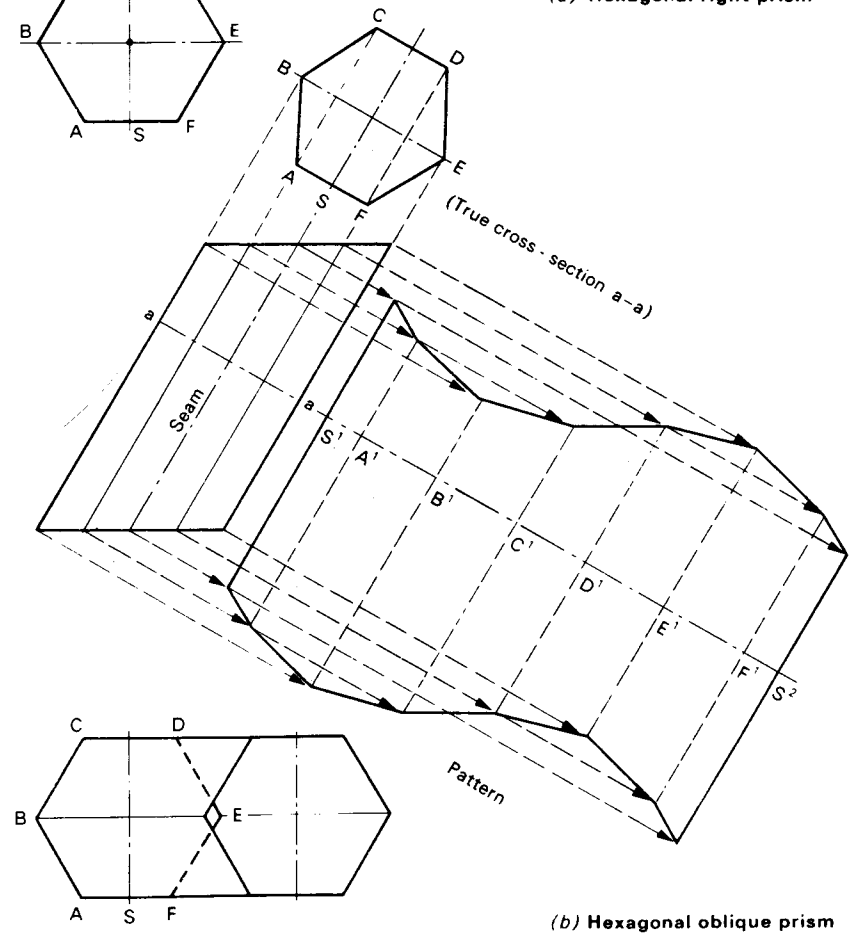


(b) Square oblique prism

Fig. 2.51 Developments of square prisms (parallel line)

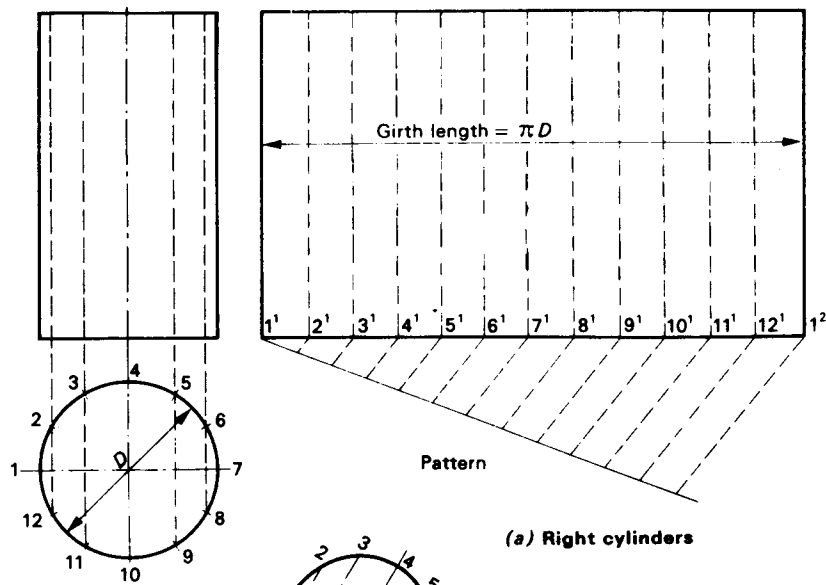


(a) Hexagonal right prism

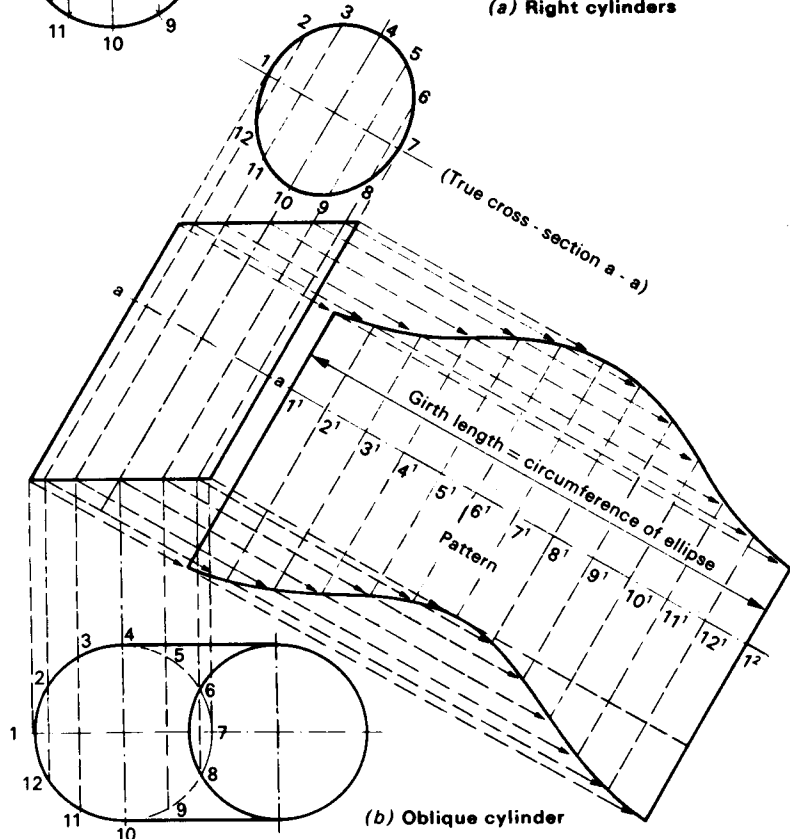


(b) Hexagonal oblique prism

Fig. 2.52 Developments of hexagonal prisms (parallel line)



(a) Right cylinders



(b) Oblique cylinder

Fig. 2.53 Developments of cylinders (parallel line)

In order to distinguish the basic difference between a 'right prism' and an 'oblique prism' it is essential to apply the basic rule:

IF THE CROSS-SECTION OF THE ENDS BETWEEN PARALLEL PLANES IS AT  $90^\circ$  TO THE AXIS, THE COMPONENT IS A RIGHT PRISM.

IF THE CROSS-SECTION OF THE ENDS BETWEEN PARALLEL PLANES IS NOT NORMAL TO THE AXIS, THE COMPONENT IS AN OBLIQUE PRISM.

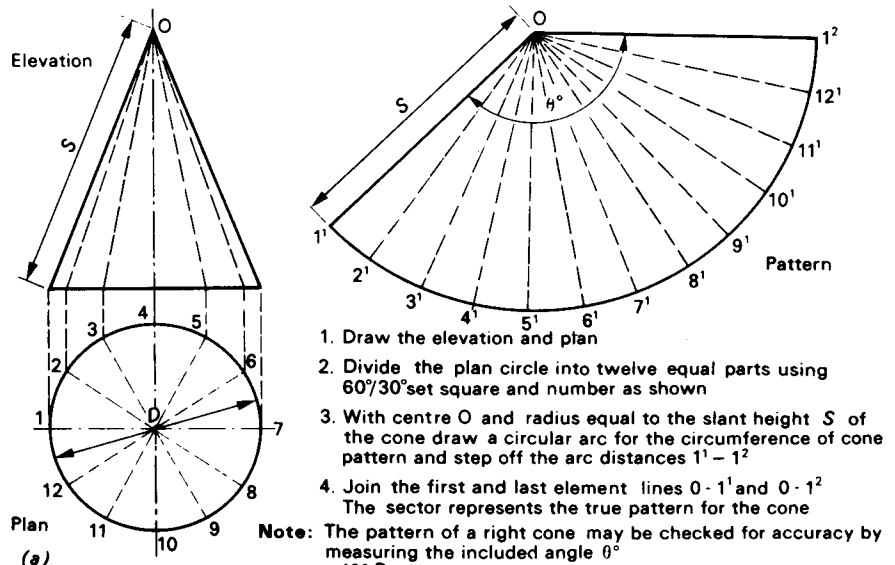
Therefore in order to develop the correct 'stretch-out' for the length of the pattern the distance around the TRUE CROSS-SECTION must be used. The examples illustrated should be self-explanatory.

## 2.17 Radial line development

The 'radial line method' may be applied for developing the pattern of any article or component which tapers to an Apex. This method is also adaptable to the development of 'frustums' which would normally taper to an apex if the sides are produced.

The principle of radial line development is based on the location of a series of lines which radiate down from the apex along the surface of the component to a base, or an assumed base, from which a curve may be drawn whose perimeter is equal in length to the perimeter of the base.

Elementary examples of radial line development are illustrated in Figs 2.54 to 2.57 inclusive.

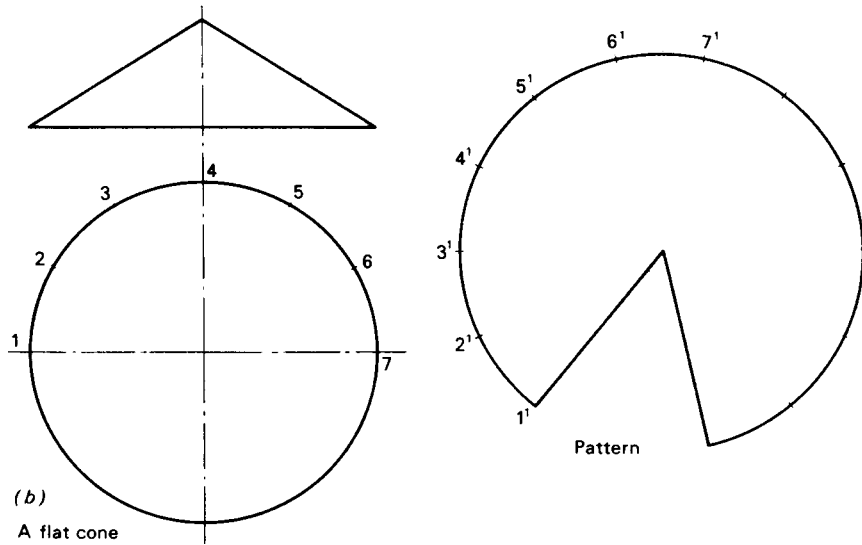


1. Draw the elevation and plan
2. Divide the plan circle into twelve equal parts using  $60^\circ/30^\circ$  set square and number as shown
3. With centre O and radius equal to the slant height  $S$  of the cone draw a circular arc for the circumference of cone pattern and step off the arc distances  $1' - 12'$
4. Join the first and last element lines  $0 - 1'$  and  $0 - 12'$   
The sector represents the true pattern for the cone

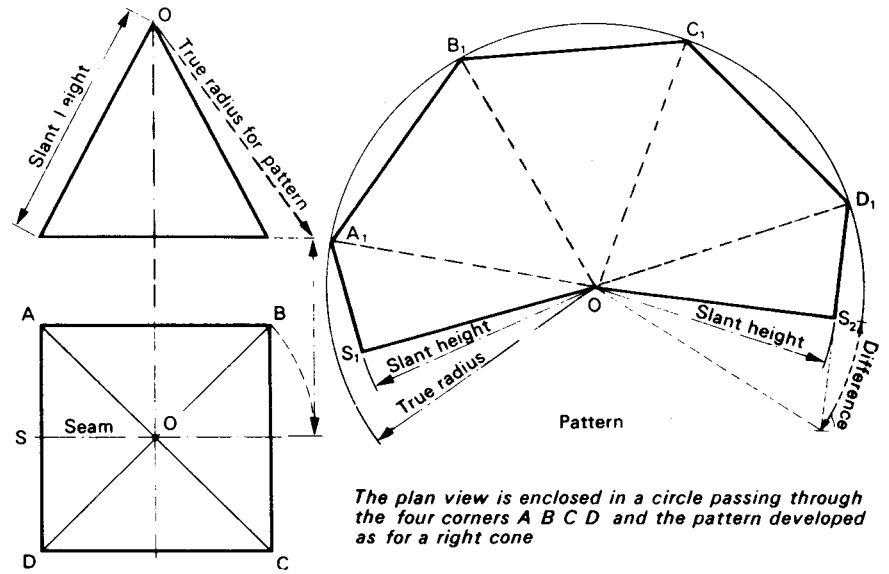
Note: The pattern of a right cone may be checked for accuracy by measuring the included angle  $\theta^\circ$

$$\theta^\circ = \frac{180D}{S} \text{ (where } D = \text{ diameter of base, } S = \text{ slant height)}$$

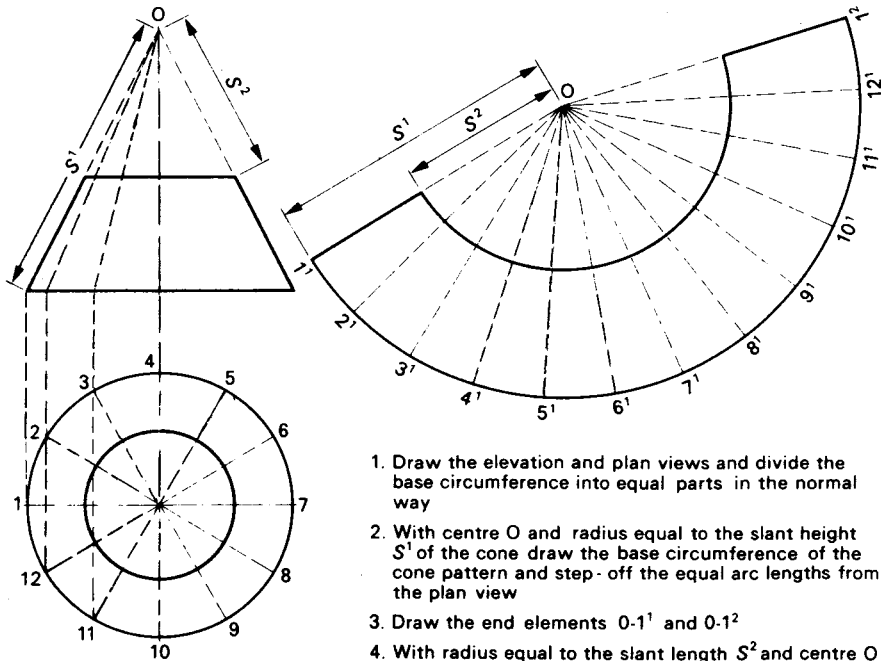
e.g. given  $D = 60\text{mm } \phi$   $S = 80\text{mm}$  then  $\theta^\circ = \frac{180 \times 60}{80} = 135^\circ$



(b)  
A flat cone  
**Fig. 2.54 Development of a right cone (radial line)**

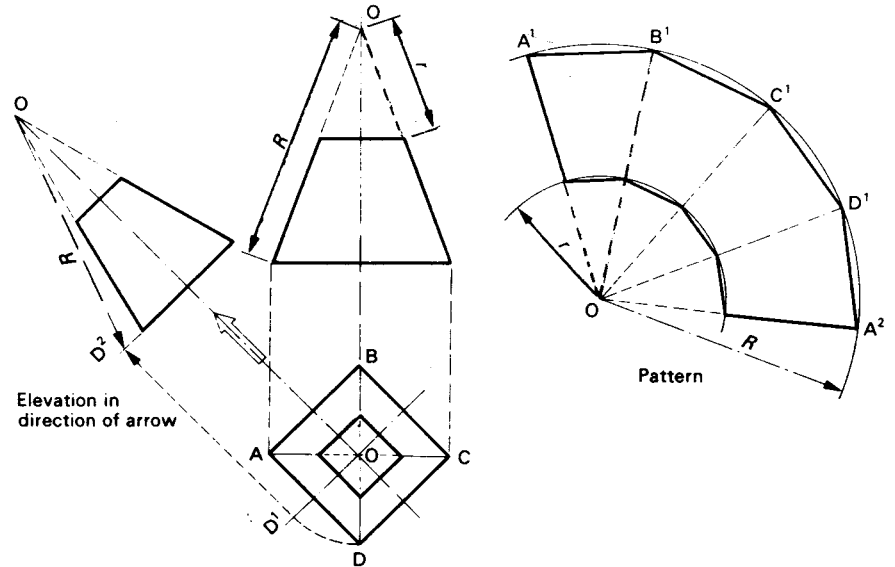


**Fig. 2.56 Development of a square - based pyramid (radial line)**



1. Draw the elevation and plan views and divide the base circumference into equal parts in the normal way
2. With centre O and radius equal to the slant height  $S^1$  of the cone draw the base circumference of the cone pattern and step-off the equal arc lengths from the plan view
3. Draw the end elements  $0-1^1$  and  $0-1^2$
4. With radius equal to the slant length  $S^2$  and centre O draw the top arc to complete the pattern

**Fig. 2.55 Development of a right cone frustum (radial line)**



**Fig. 2.57 Development of a square-based pyramid frustum (radial line)**

Cones and pyramids are very closely related geometrical shapes. *A pyramid may be considered as 'a cone with a limited number of sides'. Similarly a cone may be considered as 'a pyramid of an infinite number of sides'. In practice, many large conical shapes in heavy gauge metal are often formed on the press brake as if they were many-sided pyramids.*

Although cones and pyramids have very similar characteristics, care must be taken when developing patterns for pyramids. It is very important to recognise one specific difference between a cone and a pyramid in order to avoid mistakes in development.

Figure 2.56 shows two views of a right pyramid which completely describe the object. The elevation shows the true slant height of faces of each triangular face which are square to the plan view. However, *the slant corners of the pyramid in the plan view are not normal to the elevation.* In order to establish their true length for the pattern, the plan view would have to be rotated until one slant corner was square to the elevation. This is not possible on the drawing board, but an arc may be drawn in plan as shown in the figure (radius O B) to the centre line and the point projected up to the base of the elevation view. The distance from the apex to this point will provide the true radius for swinging the arc for the basis of the pattern. It will also be noticed, in this example, that the seam is to be along the centre of one face of the pyramid. Therefore the true length of the joint line is equal to the slant height shown in the elevation. The three full sides are marked off along the basis curve in the pattern, in the same manner as for a right cone, an arc is swung each end using centre O and radius equal to the slant height, and the last two triangles are completed by swinging arcs from A and D with a radius equal to half the length of one side in the plan. The fundamental difference between the cone and pyramid is clarified in Fig. 2.57.

## 2.18 Development by triangulation

Triangulation is by far the most important method of pattern development since a great number of fabricated components transform from one cross-section to another. A typical 'square-to-round' transformer is illustrated in Chapter 5. The basic principle of triangulation is to develop a pattern by dividing the surface of the component into a number of triangles, determine the true size and shape of each, and then lay them down side by side in the correct order to produce a pattern.

*To obtain the true size of each triangle, the true length of each side must be determined and then placed in the correct relationship to the other sides.*

### THE GOLDEN RULE OF TRIANGULATION:

'PLACE THE PLAN LENGTH OF A LINE AT RIGHT ANGLES TO ITS VERTICAL HEIGHT, THE DIAGONAL WILL REPRESENT ITS TRUE LENGTH.'

An elementary example of the method of triangulation is shown in Fig. 2.58.

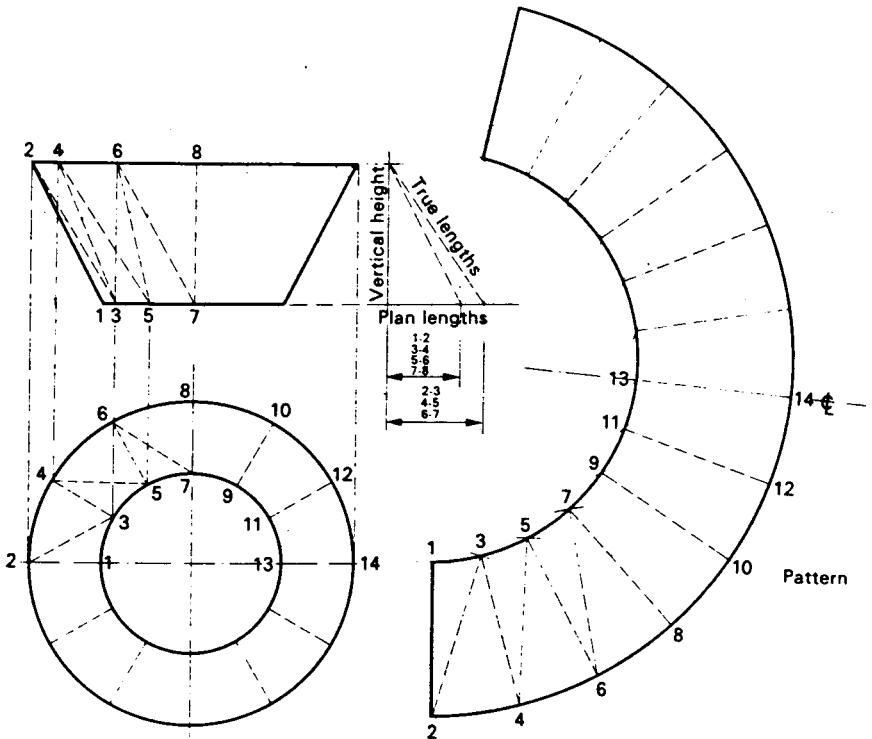


Fig 2.58 Development of a truncated cone (triangulation)

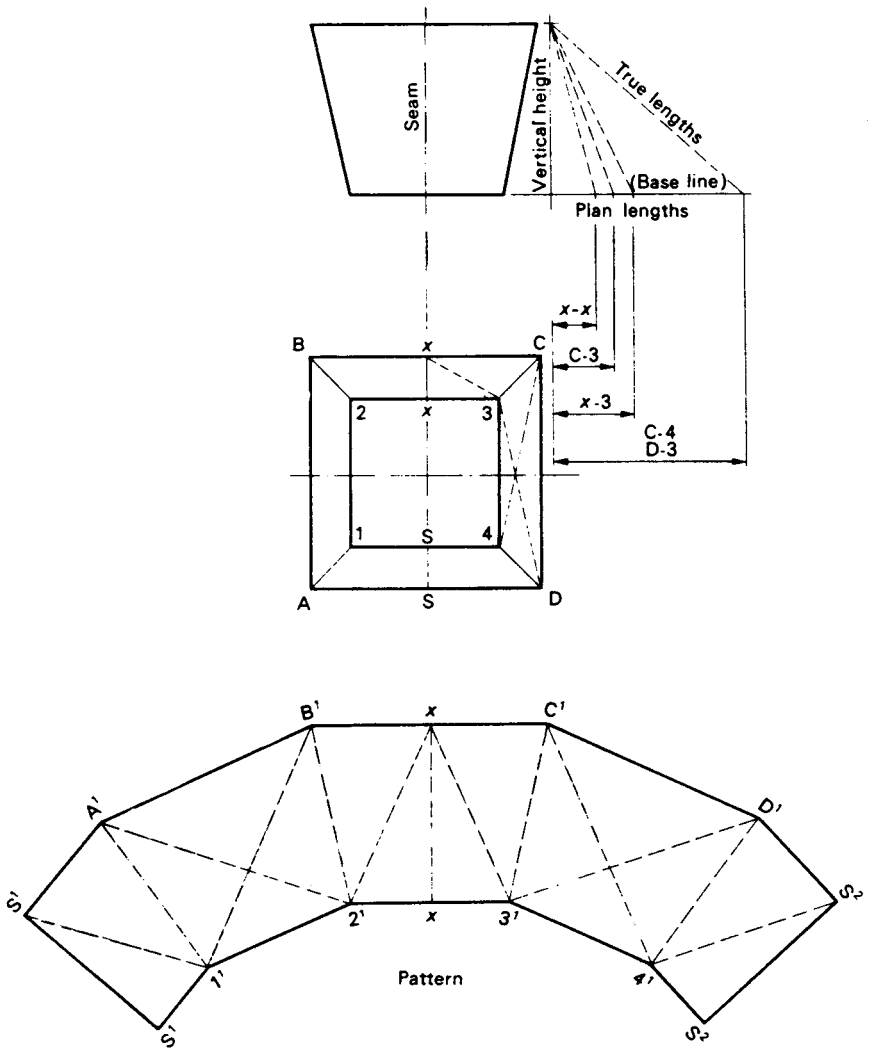


Fig 2.59 Development of a square hopper (triangulation)

- A more complex example of triangulation is shown in Fig. 2.59.
1. Draw the elevation and plan views. The corner points in plan are lettered A B C D and numbered 1, 2, 3, 4 and 5,5 denotes the seam.

2. It will be seen that the lengths A B, B C, C D and D A of the large square, and lengths 1,2, 2,3, 3,4 and 4,1 of the small square are TRUE LENGTHS in plan since they lie in the same horizontal plane, and therefore have no vertical height.
3. For the first triangle in the pattern take the true length distance B C (in plan) and mark it in the pattern. Draw a vertical centre line  $x,x$ . Mark the plan length  $x,x$  along the base line at  $90^\circ$  to the vertical height, and obtain its TRUE LENGTH and mark it on the pattern. Obtain the true length of diagonal  $x,3$  in plan and swing arcs from  $x$  (on B C) in the pattern. Complete the triangle in the pattern by taking true length  $x,3$  (in plan) and swing arcs each side of the centre line to locate 2 and 3.
4. Join B,2 and C,3 in the pattern (this represents one side of the hopper) check these two sides by plotting plan length C-3 against the vertical height.
5. For the next triangle mark true length arc B A. Obtain the true lengths of diagonal 2,A in plan by plotting it against the vertical height and swing an arc from 2 in the pattern to locate point A. Join B,A in the pattern.
6. For the next triangle swing true length arc 2,1 and true length arc B,1 these will intersect to locate point 1 in the pattern. Join A,1 and 2,1 to complete a second side of the hopper.
7. Take 3,1 in plan and swing an arc from 1 in the pattern. Take true length A,S from the plan view and swing an arc from A in the pattern to obtain true length 1,S by plotting its plan length against the vertical height and swing an arc from 1 in the pattern to locate points S. The last triangle S,1,S is completed by swinging an arc from S equal to the true length of the front line. Join A,S, 1,S and S,S.

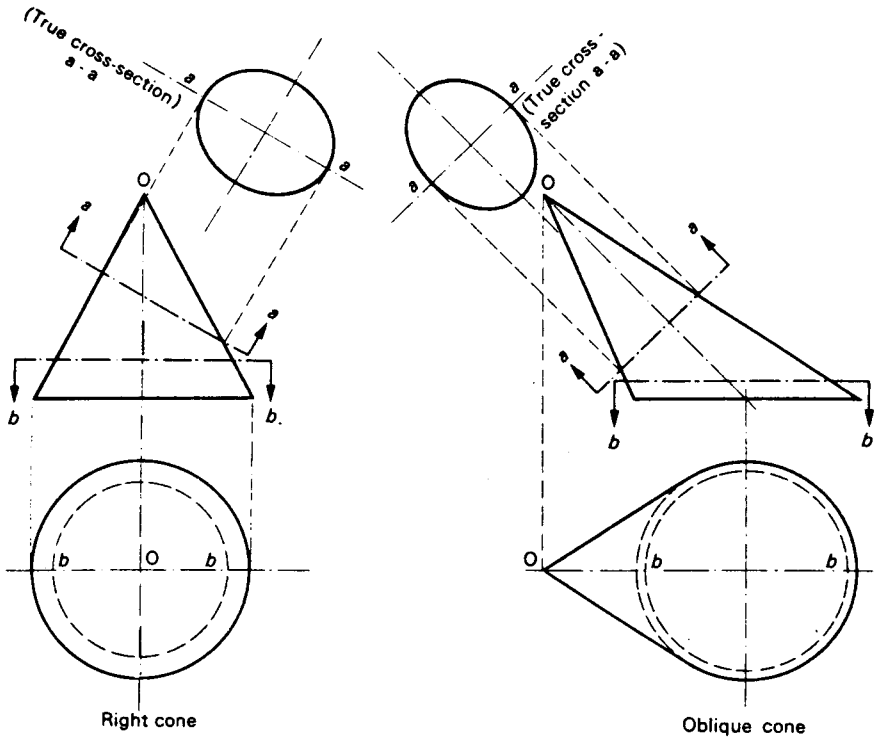
**Note:** By commencing the pattern in the middle at  $x-x$  (i.e. opposite the seam). The whole pattern to be obtained by repeating the marking out procedure each side of the centre line.

Check the pattern for symmetry — if drawn correctly the last two triangles are right-angle triangles.

## 2.19 Comparison of right and oblique cones

Although the development of oblique cones and their frustums is beyond the scope of this book, it is important to be able to recognize the essential differences between these very similar geometrical shapes.

The essential differences between right and oblique cones is explained in Fig. 2.60.



**Note:**

When an oblique cone is cut by a plane normal to its axis the cross-section produced is a true ellipse

Whereas, in the case of the right cone, the distance from the apex to the base is the same at all positions round the surface, the oblique cone varies from point to point

**Fig. 2.60 Comparison of right and oblique cones**



A RIGHT CONE has a circular base and its apex lies perpendicularly over the centre of its base.

AN OBLIQUE CONE has a circular base, but its apex does not lie perpendicularly over the centre of its base. The axis of an oblique cone leans to one side of the perpendicular. Both cones when cut by a plane parallel to the base (as shown at  $b,b$ ) present a true circle at the plane of cutting. In the case of the right cone a cutting plane parallel to its base is normal to its axis. A right cone cut obliquely presents an elliptical cross-section (as shown at  $a,a$ ).

Figure 2.61 shows typical frustums of oblique cones.

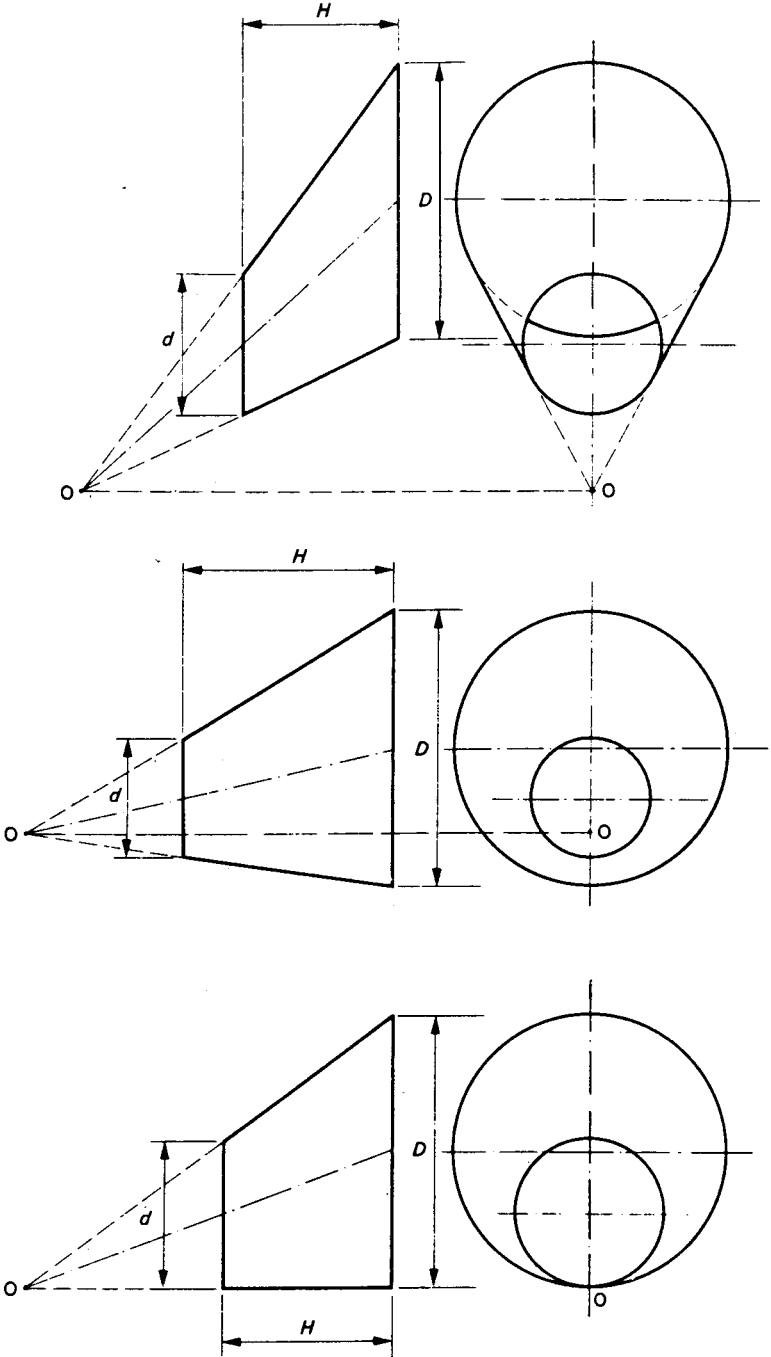


Fig 2.61 Typical oblique cone frustums